



# Levi L. Conant Prize

This prize was established in 2000 in honor of Levi L. Conant to recognize the best expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Levi L. Conant (1857–1916) was a mathematician who taught at Dakota School of Mines for three years and at Worcester Polytechnic Institute for twenty-five years. His will included a bequest to the AMS effective upon his wife's death, which occurred sixty years after his own demise.



Andrej Bauer

## Citation

### Andrej Bauer

Andrej Bauer, a professor of computational mathematics at the University of Ljubljana in Slovenia, will receive the 2022 AMS Levi L. Conant Prize for the article, "Five stages of accepting constructive mathematics," *Bulletin of the AMS*, 54 (2017), 481–498.

Bauer's article is an introduction to constructive mathematics. He begins by recalling the

basic premise of constructive mathematics—namely, that one gives up the law of *excluded middle*—then quickly clears up several common confusions. He explains Diaconescu's result that the axiom of choice implies excluded middle before proving that excluded middle is equivalent to the statement that subsets of finite sets (in the constructivist sense) are finite. The article gently introduces some of the basic ideas of constructivism. Finally, Bauer leads the reader on the first few baby steps of doing constructivist mathematics, where subtle changes in the statement of a theorem (otherwise indistinguishable from a non-constructivist viewpoint) make a significant difference. For example, while the usual formulation of the intermediate value theorem is no longer valid, it is true that a continuous function  $f : [0;1] \rightarrow \mathbb{R}$  such that  $f(x) < 0$  or  $f(x) > 0$  for every  $x \in [0,1]$  is either everywhere positive or everywhere negative.

Most expository articles in mathematics are written under the reasonable assumption that the reader is interested and receptive to the material under discussion. Many mathematicians, however, display a skepticism or even hostility towards constructivism. Bauer's article directly

addresses such readers, and takes them on a guided tour through the garden of constructive mathematics. While most mathematicians are unlikely to fundamentally change the way they do mathematics after reading this article, there are very few who will not have their eyes opened to a way of thinking they may not have otherwise appreciated—even on issues they thought they already understood. This is all accomplished with a deft sense of humor and patience.

Intuitionistic (constructivist) logic is important in type theory and programming languages, because it arises naturally there. It would have been easier for Bauer to lean on these applications as justification for considering constructivism. But Bauer writes with the confidence of someone who knows that ideas coming out of constructivist mathematics are interesting tout court, regardless of the reader's philosophical dispositions. This is an article whose ideas will stay with the reader long after it has been read.

## Biographical Note

**Andrej Bauer** is a professor of computational mathematics at the Faculty of Mathematics and Physics of the University of Ljubljana. In 1994 he received his ScB in mathematics from Ljubljana, and in 2000 he received his PhD in pure and applied logic from Carnegie Mellon University under the supervision of Dana S. Scott. In 2001, he spent a semester at the Mittag-Leffler Institute in Stockholm, Sweden. In 2012, he was a fellow at the Institute for Advanced Study, where he contributed to the development of homotopy type theory. Bauer's work spans foundations of mathematics, constructive and computable mathematics, type theory, homotopy type theory, and mathematical principles of programming languages. He is an author of the book *Homotopy Type Theory: Univalent Foundations of Mathematics* and the initiator of the HoTT library, an extensive formalization of homotopy type theory in the Coq proof assistant. He

is also known for his seminal work on programming with algebraic effects and handlers. Lately he has been working on type theory and the design of proof assistants.

Bauer's interests outside mathematics include computer-generated art and aikido, in which he holds a master's degree. He is married to Mili Bauer and they have two children, Julija and Jure.

### **Response from Andrej Bauer**

I am truly honored and grateful to receive the 2022 Levi L. Conant Prize. I thank the AMS for recognizing the effort invested into the article, and the editor Mark Goresky for kind and sustained encouragement that kept me going until the article took an acceptable form. This is also a good opportunity to state my gratitude to the small but resilient constructive mathematics community, which has given me far more than I could ever give back.

I first came into contact with constructive mathematics during my graduate years, while studying computability in analysis and topology. I still remember how difficult it was to learn constructive thinking and to suppress the instincts distilled into me by classical mathematical training. When I became a teacher, I enjoyed explaining constructive mathematics to students, colleagues, and strangers on the Internet. With time, I got better at helping them overcome common stumbling blocks and purge misconceptions that impeded their intuitions. The article, "Five stages of accepting constructive mathematics," is the synthesis of these experiences, as well as an honest disclosure of my personal views on constructive mathematics and mathematics in general.

### **Credits**

Photo of Andrej Bauer is courtesy of Marko Cokan – Foto Kozama.