The *exact* asymptotic behavior of the exchange-energy density of any finite many-electron system is known, and is given by

$$\lim_{r \to \infty} U_X^{\sigma} = -\frac{1}{r} \tag{1}$$